AIMMS Modeling Guide - Media Selection Problem

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Chapter 9

A Media Selection Problem

This chapter

This chapter introduces a simplified media selection problem and formulates it as a binary programming model. An initial model is extended to include various strategic preference specifications and these are implemented by adding logical constraints. The problem is illustrated using a worked example and its integer solutions are reported. At the end of the chapter the problem is described as a *set covering problem*. The two related binary models of *set partitioning* and *set packing models* are also discussed in general terms.

References

Examples of media selection problems are found in the marketing and advertising literature. Two references are [Ba66] and [Ch68].

Keywords

Integer Program, Logical Constraint, Worked Example.

9.1 The scheduling of advertising media

Media selection problems

Optimization is used in the field of marketing to optimally allocate advertising budgets between possible advertising outlets. These problems are known as media selection problems.

Problem description

Consider a company which wants to set up an advertising campaign in preparation for the introduction of a new product. Several types of audiences have been identified as target audiences for the new product. In addition, there is a selection of media available to reach the various targets. However, there is no medium that will reach all audiences. Consequently, several media need to be selected at the same time in order to cover all targets. The company wants to investigate various strategic advertising choices. The goal is *not* to stay within an a priori fixed budget, but to minimize the total cost of selecting media for each of the strategic choices.

Example

This chapter illustrates the problem using a small data set involving six target audiences (labeled type 1 through type 6) and eight potential medias. The data is contained in Table 9.1. The media descriptions are self-explanatory. The crosses in the table indicate which target audiences can be reached by each particular medium. Note that a cross does not say anything about the effectiveness of the medium. The right hand column gives the cost to use a particular medium. The table is deliberately small for simplicity reasons. In practical applications both the data set and the reality behind the scheduling problem is more extensive.

media	**audience type 1**	**type 2**	**type 3**	**type 4**	**type 5**	**type 6**	**costs [$]**
Glossy magazine	×			×			20,000
TV late night		×	×				50,000
TV prime time		×				×	60,000
Billboard train	×					×	45,000
Billboard bus			×				30,000
National paper				×		×	55,000
Financial paper		×			×		60,000
Regional paper	×				×		52,500

Table 9.1: Reachability of audiences by media

9.2 Model formulation

Verbal model

The aim is to construct a model to determine which media should be selected so that all audiences are reached. It does not matter if an audience is covered more than once, as long as it is covered at least once. Moreover, the company does not wish to spend more money on the campaign than necessary. The objective function and constraints are expressed in the following qualitative model formulation:

Minimize: *total campaign costs,*
Subject to:
for all audience types: the number of times an audience type is covered must be greater than or equal to one.

Notation

The above verbal model statement can be specified as a mathematical model using the following notation.

Indices:

- t — *target audiences*
- m — *advertising media*

Parameters:

- N_{tm} — *incidence: audience t is covered by medium m*
- c_m — *cost of selecting advertising medium m*

Variables:

x_m *binary, indicating whether advertising medium m is selected*

Covering constraint

Advertising media should be selected to ensure that all audiences are reached at least once. This is guaranteed by the following covering constraint.

$$\sum_m N_{tm} x_m \geq 1 \qquad \forall t$$

Objective function

The objective function is to minimize the cost of covering all target audiences at least once.

Minimize:

$$\sum_m c_m x_m$$

Model summary

The following mathematical statement summarizes the model.

Minimize:

$$\sum_m c_m x_m$$

Subject to:

$$\sum_m N_{tm} x_m \geq 1 \qquad \forall t$$

$$x_m \in \{0, 1\} \qquad \forall m$$

The problem is a binary programming model since all decision variables are binary. Using the terminology introduced in Chapter 2.2, it is also a zero-one programming problem.

Model results

The small model instance provided in this chapter can easily be solved using conventional integer programming code. Table 9.2 provides the solution values for both the integer program and the linear program. In the case of the latter solution, unlike in Chapter 8, it does not make sense to round up or down. The cost of the campaign amounts to $155,000 for the integer solution, and $150,000 for the (unrealistic) linear programming solution. Note that the audience of type 1 is covered twice in the integer solution, while all other audiences are reached once.

Advertising media	x_{IP}	x_{LP}
Glossy magazine	1	0.5
TV late night		
TV prime time		
Billboard train	1	0.5
Billboard bus	1	1.0
National paper		0.5
Financial paper	1	1.0
Regional paper		

Table 9.2: Optimal solution values for integer and linear program

9.3 Adding logical conditions

Logical constraints

Logical relationships between different decisions or states in a model can be expressed through logical constraints. In the media selection problem, logical constraints can be imposed relatively simply because the decision variables are already binary. Some modeling tricks for integer and binary programming model were introduced in Chapter 7. This section provides some additional examples of modeling with logical conditions.

Must include television commercials

Suppose the marketing manager of the company decides that the campaign should, in all cases, incorporate some TV commercials. You can model this condition as follows.

$$x_{\text{TV late night}} + x_{\text{TV prime time}} \geq 1$$

This constraint excludes the situation where both $x_{\text{TV late night}}$ and $x_{\text{TV prime time}}$ are zero. When this constraint is added to the model, the optimal solution includes late night TV commercials as well as advertisements in national and regional newspapers for the advertising campaign. The campaign costs increase to $157,500.

If billboard then television

Suppose that if a billboard media is selected, then a television media should also be selected. Perhaps the effects of these media reinforce each other. A precise statement of this condition in words is:

- *If at least one of the billboard possibilities is selected, then at least one of the possibilities for TV commercials must be selected.*

The following AIMMS constraint can be used to enforce this condition.

$$x_{\text{TV late night}} + x_{\text{TV prime time}} \geq x_{\text{Billboard train}}$$
$$x_{\text{TV late night}} + x_{\text{TV prime time}} \geq x_{\text{Billboard bus}}$$

Note that these inequalities still allow the inclusion of TV commercials even if no billboard medias are selected.

Billboard if and only if television

Next, consider the following condition which imposes a one-to-one relationship between billboards and television.

- *If at least one of the billboard possibilities is selected, then at least one of the possibilities for TV commercials must be selected,* and *if at least one of the possibilities for TV commercials is selected, then at least one of the billboard possibilities must be selected.*

As this condition consists of the condition from the previous section plus its converse, its formulation is as follows.

$$\begin{aligned} x_{\text{TV late night}} + x_{\text{TV prime time}} &\geq x_{\text{Billboard train}} \\ x_{\text{TV late night}} + x_{\text{TV prime time}} &\geq x_{\text{Billboard bus}} \\ x_{\text{Billboard train}} + x_{\text{Billboard bus}} &\geq x_{\text{TV late night}} \\ x_{\text{Billboard train}} + x_{\text{Billboard bus}} &\geq x_{\text{TV prime time}} \end{aligned}$$

After solving the model with these inequalities, the glossy magazine, TV commercials at prime time, billboards at bus-stops, and advertisements in regional newspapers are selected for the campaign. The campaign cost has increased to $162,500. Just like the initial integer solution, the audience of type 1 has been covered twice.

If television prime time then no billboards

Consider a condition that prevents the selection of any billboard media if prime time TV commercials are selected. A verbal formulation of this condition is:

- *If TV commercials at prime time are selected then no billboards should be selected for the campaign.*

Note that, where the previous inequalities implied the selection of particular media, this condition excludes the selection of particular media. The above statement can be modeled by adding a single logical constraint.

$$x_{\text{Billboard train}} + x_{\text{Billboard bus}} \leq 2(1 - x_{\text{TV prime time}})$$

Note that if $x_{\text{TV prime time}}$ is equal to 1, then both $x_{\text{Billboard train}}$ and $x_{\text{Billboard bus}}$ must be 0. Adding this constraint to the media selection model and solving the model yields an optimal integer solution in which the glossy magazine, late night TV commercials, billboards at railway-stations, and advertisement in regional newspapers are selected for the campaign. The corresponding campaign cost increase to $167,500.

Suppose that the marketing manager wants the financial paper to be included in the campaign whenever both late night TV commercials and the glossy magazine are selected. The condition can be stated as follows.

If late night television and magazine then financial paper

- *If late night TV commercials and the glossy magazine are selected then the financial paper should be selected for the campaign.*

This condition can be incorporated into the model by adding the following logical constraint.

$$x_{\text{Financial paper}} \geq x_{\text{TV late night}} + x_{\text{Glossy magazine}} - 1$$

Note that this constraint becomes $x_{\text{Financial paper}} \geq 1$ if both $x_{\text{TV late night}}$ and $x_{\text{Glossy magazine}}$ are set to 1. After adding this constraint to the model, the advertisements in regional newspapers from the previous solution are exchanged for advertisements in the financial paper, and the corresponding campaign cost increases to \$175,000. Now, audiences of type 1 and 2 are covered twice.

The final extension to the model is to add a constraint on the audiences. In the last solution, the number of audiences that are covered twice is equal to two. The marketing manager has expressed his doubts on the reliability of the reachability information, and he wants a number of audience types to be covered more than once. Specifically, he wants the following.

At least three audiences should be covered more than once

- *At least three audience types should be covered more than once.*

To formulate the above logical requirement in mathematical terms an additional binary variable y_t is introduced for every audience type t. This variable can only be one when its associated audience t is covered more than once. The sum of all y_t variables must then be greater than or equal to three. Thus, the model is extended with the following variables and constraints.

$$\begin{aligned} 2y_t &\leq \textstyle\sum_m N_{tm} x_m \qquad \forall t \\ \textstyle\sum_t y_t &\geq 3 \\ y_t &\in \{0, 1\} \end{aligned}$$

Note that the expression $\sum_m N_{tm} x_m$ denotes the number of times the audience of type t is covered, and must be at least two for y_t to become one. When solving the media selection model with this extension, all media except prime time TV commercials and advertisements in the national paper and the regional papers are selected. The audiences of type 1, 2 and 3 are covered twice, and the total campaign cost is \$205,000.

9.4 Set covering and related models

Set covering

The media selection problem can be considered to be a *set covering* problem. A general statement of a set covering problem follows. Consider a set $S = \{s_1, s_2, \ldots, s_n\}$ and a set of sets U which consists of a number of subsets of S. An example would be

$$S = \{s_1, s_2, s_3, s_4, s_5, s_6\} \text{ and}$$
$$U = \{u_1, u_2, u_3, u_4\} = \big\{\{s_1, s_2\}, \{s_3, s_4, s_6\}, \{s_2, s_3, s_5\}, \{s_5, s_6\}\big\}$$

Let each of these subsets of S have an associated cost, and consider the objective to determine the least-cost combination of elements of U such that each element of S is contained in this combination at least once. Every combination which contains each element of S is called *a cover*. In this example, $\{u_1, u_2, u_4\}$, $\{u_1, u_2, u_3\}$ and $\{u_1, u_2, u_3, u_4\}$ represent the only covers. It is not difficult to determine the least expensive one. However, when the sets S and U are large, solving an integer program becomes a useful approach.

Notation

In order to specify an appropriate model, the binary decision variable y_u must be defined.

$$y_u = \begin{cases} 1 & \text{if } u \in U \text{ is part of the cover} \\ 0 & \text{otherwise} \end{cases}$$

Furthermore coefficients a_{su} must be introduced.

$$a_{su} = \begin{cases} 1 & \text{if } s \in S \text{ is contained in } u \in U \\ 0 & \text{otherwise} \end{cases}$$

The integer program

When the costs are defined to be c_u for $u \in U$, the model statement becomes:

Minimize:

$$\sum_{u \in U} c_u y_u \qquad \text{(combination costs)}$$

Subject to:

$$\sum_{u \in U} a_{su} y_u \geq 1 \qquad \forall s \in S$$
$$y_u \quad \textit{binary} \qquad \forall u \in U$$

Note that all constraint coefficients and decision variables have a value of zero or one. Only the cost coefficients can take arbitrary values. For the special case of uniform cost coefficients, the objective becomes to minimize the number of members of U used in the cover.

The set partitioning problem

When all elements of S must be covered exactly once, the associated problem is termed a *set partitioning problem*. As the name suggests, the set S must now be partitioned at minimum cost. The corresponding integer programming model is similar to the above model, except for the signs of the constraints. These are "=" rather than "≥".

The set packing problem

When all elements of S can be covered at most once, the associated problem is termed a *set packing problem*. The corresponding integer programming model is similar to the model stated previously, except for two changes. The signs of the constraints are "≤" rather than "≥", and the direction of optimization is "maximize" instead of "minimize".

Applications

There are several applications which can be essentially classified as covering, partitioning or packing models.

1. If audience types are considered to be members of the set S, and advertising media members of the class U, you obtain the media selection problem which is an example of set covering.
2. Consider an airline crew scheduling problem where flights are members of S, and "tours" (combinations of flights which can be handled by a single crew) are members of the set U. Then, depending on whether crews are allowed to travel as passengers on a flight, either a set covering or a set partitioning model arises.
3. Let the set S contain tasks, and let the set U contain all combinations of tasks that can be performed during a certain period. Then, if each task needs to be performed only once, a set partitioning problem arise.
4. Finally, if the set S contains cities, and the class U contains those combinations of cities that can be served by, for instance a hospital (or other services such as a fire department or a university), then a set covering model can determine the least cost locations such that each city is served by this hospital.

9.5 Summary

In this chapter a media selection problem was introduced and formulated as a binary programming model. An initial model was extended by including a variety of logical constraints to represent various advertising strategies. The optimal objective function value and corresponding integer solution were reported for each subsequent model. At the end of the chapter, the media selection problem was described as a *set covering problem*. The related *set partitioning* and *set packing* problems were discussed in general terms.

Exercises

9.1 Implement the initial mathematical program described in Section 9.2 using the example data of Table 9.1. Solve the model as a linear program and as an integer program, and verify that the optimal solutions produced with Aimms are the same as the two optimal solutions presented in Table 9.2.

9.2 Extend the mathematical program to include the logical constraints described in Section 9.3, and verify that the objective function values (the total campaign cost figures) produced with Aimms are the same as the ones mentioned in the corresponding paragraphs.

9.3 Formulate the following requirements as constraints in Aimms.

- If at least one of the billboard possibilities is selected, then both of the possibilities for TV commercials must be selected.
- At least five of the six audience types need to be covered.
- Again, at least five of the six audience types need to be covered. If, however, not all six audience types are covered, then either the regional paper or the national paper should be selected.

Develop for each requirement a separate experiment in which you either modify or extend the initial mathematical program described in Section 9.2. Verify for yourself that the integer solution correctly reflects the particular requirement.

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