AIMMS Modeling Guide - Background

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AIMMS B.V.
Diakenhuisweg 29-35
2033 AP Haarlem
The Netherlands
Tel.: +31 23 5511512

AIMMS Inc.
11711 SE 8th Street
Suite 303
Bellevue, WA 98005
USA
Tel.: +1 425 458 4024

AIMMS Pte. Ltd.
55 Market Street #10-00
Singapore 048941
Tel.: +65 6521 2827

AIMMS
SOHO Fuxing Plaza No.388
Building D-71, Level 3
Madang Road, Huangpu District
Shanghai 200025
China
Tel.: ++86 21 5309 8733

Email: info@aimms.com
WWW: www.aimms.com

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Part I

Introduction to Optimization Modeling

Chapter 1

Background

This chapter

This chapter gives a basic introduction to various aspects of modeling. Different types of models are distinguished, some reasons for using models are listed, some typical application areas of modeling are mentioned, and the roles of mathematics and AIMMS are described. Finally, an overview is given of the "model formulation process."

1.1 What is a model?

A model

Since this guide describes different models and modeling techniques, it will start by defining what a model is. In general: *a model is a prototype of something that is real.* Such a prototype can be concrete or abstract.

Concrete model

An example of a concrete model is a teddy bear. A teddy bear is concrete—you can touch it and play with it—and it is also a scaled-down (and somewhat friendlier) version of a real bear.

Abstract model

To illustrate an abstract model, the Teddy Bear Company is introduced. This company produces black and brown teddy bears in three sizes, and its owners consider the teddy bear in terms of an *abstract* model. That is, they describe everything they need to know about producing it:

- materials: fur cloth in black and brown, thread, buttons, different qualities of foam to stuff the bears,
- information on prices and suppliers of materials,
- three different sized patterns for cutting the fur, and
- assembly instructions for the three sizes of bears.

In this abstract model of the teddy bear, there is just the basic information about the bear. A *mathematical model* can be used to derive further information.

Mathematical models

Suppose you want to know the cost of a small black teddy bear. Then you sum the material costs (material prices by the amounts used) and the production costs (assembly time by labor cost). The relationship between all these components can be expressed in general terms. Once formulated, you have developed a mathematical model. A mathematical model is an abstract model. It is a description of some part of the real world expressed in the language of mathematics. There may be some variables in the description—values that are unknown. If the model is formulated so that you can calculate these values (such as the cost of a small black teddy bear) then you can *solve* the model.

Optimization models

One class of mathematical models is referred to as *optimization models*. These are the main subject of this guide. For now, a small example will be given. Recall that the teddy bears are filled with foam which comes in small pieces and is available in different qualities. By choosing a certain mix you can achieve a specified "softness" for each size of teddy bear. The company wants to decide how much to buy of each quality. A constrained optimization model could be used to determine the cheapest mix of foams to yield a certain softness. Note that the *optimization* is in the determination of the *cheapest* combination (cost minimization), and that the constraint(s) are in terms of the softness requirement.

1.2 Why use models?

This section

In Section 1.1 two types of models were introduced and some typical modeling applications given. Advantages of using models are now listed.

Learning while constructing

A model serves as a learning tool. In the process of constructing a model you are forced to concentrate on all the elements required to realize the model. In this way, you learn about the relationships between the elements that make up the model. For instance, during the design of a teddy bear you must consider the outside—what is the shape of the ears?—as well as the inside—the foam to choose. During the process, you may discover an unexpected relationship between the softness of the bear and its assembly.

Filtering information

Depending on the purpose of the model you then need to set priorities since mathematical models can grow in complexity with the quantity of detail. If the objective of the model is to determine the cheapest mix of foams, you may wish to ignore the softness of the ears. Such filtering and structuring of information is an important contribution to the modeling process.

Medium of discussion

Models can also be a valuable tool of expression during discussions between concerned parties. If the formulation and data of a model are agreed, it can be used to make predictions of the consequences of different actions, which can then be compared.

Making predictions

Some decisions are irreversible or expensive. In these cases, it is beneficial to be able to predict the consequences and to experiment with different options. Again models can help. The Teddy Bear Company may use the optimization model to calculate the cost of different foam mixes while varying the softness requirements. Perhaps a slightly softer bear turns out to be considerably cheaper.

Cutting costs to stay competitive

For the Teddy Bear Company it might be possible to cut the foam cost further by buying it from several suppliers and monitoring price variations. The optimization model could easily be extended to incorporate this information. It will enable the Teddy Bear Company to react quickly to changes in foam prices.

With computers

The combination of models and computers enhances the speed of decision making. The computer enables you to handle far more information than could be done manually.

Streamlining

Another benefit possible from modeling is called *streamlining*. That is, in the process of constructing a model and structuring its information you may discovery inefficiencies which can then be addressed. For example, consider a consulting company, while making a model of the Teddy Bear Company as a whole, discovers that there are two business units performing virtually the same task. Suppose that for some historical reason, there are separate units producing black and brown teddy bears. Since the only difference is in the color of the cloth, integrating these business units would make the company more efficient.

1.3 The role of mathematics

Mathematical requirements

Even though this guide concerns the specification of *mathematical* programs (optimization models), you do not have to be a mathematician to become a good modeler. The degree of mathematics required depends upon the sort of model you are building. In this book there is a distinction between basic, intermediate and advanced models such that each type requires an increase in knowledge of mathematical theory and concepts. When considering the role of mathematics, one can distinguish three aspects, namely, language, theory and algorithms. These aspects are discussed in the following paragraphs.

Mathematical language

Mathematics provides a language in which abstract models can be formulated using the following elements.

- Mathematical concepts such as variables (unknowns) and parameters (symbols representing known data);
- Operators such as:
  - unary operators (+, −, NOT),
  - comparison operators (equal to, not equal to, etc.),
  - algebraic operators (addition, subtraction, multiplication, division, power, etc.),
  - logical operators (AND, OR, etc.),
  - differential operators and integral operators;
- Data: which links a model to a real-world situation.

Mathematical theory

Mathematics provides a theoretical framework for the formulation and solution of those models for which the translation from problem to model is not immediate. Mathematical concepts are then needed to capture aspects of tangible and non-tangible real-life situations. A non-tangible example is the risk minimization in a financial portfolio selection model. There is no hard definition of *risk* but there are plenty of interpretations. When building a model one or more of these interpretations must be made concrete in the form of a mathematical formula for risk. This can only be achieved by using mathematical concepts and their associated theory.

Mathematical algorithms

Mathematical algorithms are needed to obtain solutions of mathematical models. Fortunately, there are algorithms that have wide spread application and give solutions for whole classes of mathematical models. As a model builder you can choose not to acquire an in-depth knowledge of these algorithms as they are commercially available and directly accessible within a modeling system. Nevertheless, there are instances of large-scale models for which standard solution algorithms no longer suffice, and special algorithms need to be developed. Examples of these can be found in Part 21 containing advanced optimization modeling applications.

AIMMS *minimizes need for mathematics*

The AIMMS modeling system is especially designed to minimize the mathematical complexity.

- The AIMMS modeling language incorporates indexing facilities to guide you in expressing and verifying complex mathematical models.
- Since there is a separation between the formulation and the *solving* of a model, you do not need to know the details of the solution method. If you do not specify a solution method, AIMMS will determine the best method for you.

1.4 The modeling process

Iteration of activities

The process of developing a model usually involves several different activities. Although these activities are listed sequentially below, several will be repeated as new information becomes available during the execution of other steps, thus an *iterative* approach will often be necessary.

- Define the goal.
- Consult literature and other people.
- Formulate the model, and collect the data.
- Initial testing.
- Validation.

Investigating contribution of a model

In a complex problem, the structure and goal of the model may not be obvious. Therefore, the first step should be to analyze the general problem conceptually and to determine which aspects of the real-world situation must be included. At this stage the emphasis is on problem definition, rather than on mathematics. It is likely there are different views on the problem and its causes, and these must be resolved so that all concerned parties agree on the goal of the model. In one sense, you can say that the goal of a model is to support a decision. In a wider sense, a model can be used to *predict* the consequences of particular decisions, and so to compare alternatives.

Investigating sources

Once the goal of the model is agreed, the next step is to investigate if a similar model has already been developed. There are benefits to review the work of others. It may help you to find the *data* you need and it may give you some hints on how to formulate your model. However, in general you can expect to do some customization.

Data collection

In most cases, by far the harder task is collecting the data. This task is both time consuming and prone to errors. Most models, built to analyze real-world problems, require lots of data. In some instances data is available through databases. In other instances data may not be readily available, and once it is found, it is seldom in a form directly suitable for your purposes. In this case data must be copied, checked for errors, and manipulated. During this *data adjustment phase*, errors of different types may be introduced. Examples are approximation errors, typographical errors, and consistency errors. Many typographical errors and inconsistencies in data can be identified by AIMMS, and will result in error messages when compiled.

Choosing a formulation

It is not usually self-evident which particular type of model formulation is the most appropriate. There are no clear rules for making a choice. As mentioned in the previous paragraph, the choices of other modelers in similar situations can give suggestions, and your own experience and knowledge will always influence your eventual choice. In any case, when you want to stay within the framework of the available modeling tools, you may have to compromise.

Initial testing

It is recommended that you start with a small model containing only the basic relationships. You should verify that these are formulated correctly before adding increased complexity, and continue in this gradual fashion.

Validation

Validation is the process of checking whether initial model results agree with known situations. It is an important final step before the model results are used in support of a real-world decision. Two situations may occur:

- A methodology already exists with the same purpose as the new model.

In order to be credible, you will have to prove that the results of the new model are *at least* as good as the results produced with an existing method. Since the "old method" is probably known to be accurate (or its weaknesses are known), you can compare the old and new methods side by side. Your model should at a minimum reproduce the old results, and you should aim for better results.

- There is no existing methodology.

In this case, you should use historical data, and try to reproduce the past. In essence, you try to predict what you already know.

1.5 Application areas

This section

This section gives an indication of the applicability of optimization models in practice. Obviously, these few pages do not pretend to give a complete overview. Rather the goal is to combine a limited overview with a description of a typical example taken from each application area mentioned. These descriptions are qualitative in that specific data and formulas are not given. The focus is on the general structure of models.

1.5.1 Production

The petroleum industry

In the petroleum industry *refinery models* are used for planning, scheduling and process control, see for instance [Ma56]. Following is a brief account of a refinery planning model.

Refinery planning model

Planning models support long-term decisions, concerning raw material purchases, and utilization of production processes, in order to meet future demands for final products with specific qualities. These quality specifications require *blending models* to be embedded in refinery models, see for instance [Ke81]. A refinery planning model describes the process from crude oils to final products such as different blends of gasoline, fuel oil, and distillate. The production process makes use of various production units including crude distillers, catalytic crackers, and vacuum distillers.

Refinery model description

This qualitative model description is based on a model as described in [Ke81].

Maximize: *profit = revenue − material cost − operating cost,*
Subject to:

- *for all crude oils: a constraint stating that the amount of crudes used in processes must not exceed the amount of crudes purchased,*
- *for all intermediates: a constraint stating that the net production and purchases of an intermediate commodity must exceed its use in blending,*
- *for each production unit: a capacity constraint,*
- *for all crude oils: limits on purchases,*
- *for all final products: a constraint stating that final sales must equal the output of the blending process, and*
- *for all final products, and all quality attributes: upper and lower bounds on the quality of final products.*

Other applications

The application area that studies the efficient production and distribution of energy is known as *energy economics*. This area is of strategic importance to modern economies because of the world's dependence on energy, and in particular oil. The conversion of scarce energy resources to electricity and heat can be modeled, see for instance [Ma76]. From such a model, the least-cost mix including extraction of energy resources, and investment in production technologies, can be determined. Moreover, constraints on emission levels for certain pollutants and the use of abatement technologies can be included. Another application is *scheduling power plants* to meet the varying load at different times of the day, and in different seasons. An example is given in Chapter 16.

1.5.2 Production and inventory management

Typical model components

The scheduling of production and inventory can be formulated as a linear optimization model. Typically, multiple time periods and material balances are incorporated in such a model. Suppose factory production must satisfy a fluctuating demand. There are many strategies for reacting to these fluctuations:

extra workers can be hired and fired, workers can work overtime, a stock inventory can be kept to cover future shortages, etc. Obviously, the hiring and firing of workers or overtime working adds extra costs. Storage however is also costly.

An example

A verbal formulation of the production and inventory problem, which can be found in [Ch83], follows.

> **Minimize:** *costs of storage, hiring, firing, and overtime work,*
> **Subject to:**
> - *for each time period: upper bounds on overtime work and changes in work force level, and*
> - *for each time period: a material balance involving production, inventory, and demand.*

Other useful references on inventory planning are [Ch55] and [Ha60].

Other applications

Linear optimization models are quite successful for *resource allocation* problems. The potato chips problem in Chapter 2 is a simplified example. Different products compete for a finite set of resources, and decisions may possibly be made about the selection of a production process for a particular product.
Another widespread application of mathematical optimization models is in developing *production schedules*. An overview of techniques is given in [Bu89].

1.5.3 Finance

Portfolio selection

A widely-used application in the finance field is the selection of *investment portfolios*. A portfolio is a collection of securities held by an investor, such as a pension fund. Diversification is the spreading of investments over different securities with the aim to reduce the overall risk content of a portfolio while still maintaining a respectable return. One of the original models is due to Markowitz ([Ma52]). An investor wants the expected return of the portfolio to be high, and the risk to be low. The risk can be measured using the statistical concept *variance*—the deviation of realized return from the expected return. Furthermore, the *covariance* between two securities is a measure of correlation. A covariance matrix of returns summarizes variance and covariance figures, and can be derived from historical data. It can be used to compute the risk attached to the objective.

An example

> **Minimize:** *risk,*
> **Subject to:**
> - *a constraint on minimum expected return,*
> - *a limitation on the budget that can be spent, and*
> - *additional constraints concerning the proportions invested in certain securities.*

The resulting model has a quadratic objective and linear constraints, and is known as a *quadratic optimization model*. In Chapter 18, a more elaborate treatment of portfolio models is given. In real situations, the size of the covariance matrix can be overwhelming. For example, 100 possible securities give rise to a covariance matrix of $100 \times 100 = 10,000$ entries. Alternative methods have been developed to compress this information.

Other applications

Optimization models can be used to plan the cash receipts and disbursements of an individual firm. These *cash management models* can sometimes be formulated as networks in which the nodes represent different time periods. The arcs then represent flows of money over time. Optimization models can also be used for the management of large funds. Such a fund requires careful planning since commitments made today have long term financial consequences. A multi-period model considers, for each period, loans, repayments, and investments, subject to financial and political constraints. In *accountancy*, linear optimization models are often used because of the useful economic information that can be derived from shadow prices and reduced costs. A thorough account of the use of optimization models in finance is given by in [Ko87].

1.5.4 Manpower planning

Scheduling work shifts

Linear and integer optimization models can be used to schedule work shifts when the demand for workers varies over the hours of the day, or the days of the week. The decision to be made is how many people should work during each shift, see for instance [Sc91] and [Ch68]. Examples are the scheduling of bus drivers, nurses, and airline crews. A general framework for the solution process is as follows:

1. Identify or make a forecast of the personnel requirements for the period to be scheduled.
2. Evaluate all possible shift patterns. All these patterns have costs associated with them, depending on efficiency, wasted time etc.
3. Determine the least cost combination of shifts that satisfies the personnel requirements.

An optimization model is used for the final step. Obviously, the second step can also be computerized. In large problems this will be essential. Advanced methods exist in which promising shift patterns are generated using shadow prices produced by the simplex method. An iterative process between the Steps 2 and 3 evolves. Such a technique is known as *delayed column generation*, see for instance [An91] and [Ch83].

A model for scheduling bus drivers

A slightly different approach is taken in the following problem. Suppose a bus company employs drivers who are entitled to two consecutive days off per week. For each day of the week the number of drivers required is known. The problem is: "How many drivers should start their five-day working week on each day of the week?" The qualitative model formulation can now be given:

Minimize: *total number of drivers,*
Subject to:
for each day: the driver requirements must be satisfied.

The formulation of the constraint is a special task, because for each day the number of drivers that are part way through their shift must also be calculated. Complicating factors might be higher costs for working during weekends, or the availability of part-time workers.

Other applications

Longer time-scale models exist which consider personnel as *human capital*, whose value increases as experience grows, training followed, etc., see for instance [Th92]. Such a model can be formulated as a *network-with-gains* model. The nodes represent moments in time, and the flow between these nodes represents the flow of employees whose value varies in time due to training, resignations, and new hiring.

1.5.5 Agricultural economics

Individual farm models

Models of both a country's agricultural sector and of individual farms have been used extensively by governments of developing countries for such purposes as assessing new technologies for farmers, deciding on irrigation investments, and determining efficient planting schedules to produce export-goods. A short account of an agricultural *subsector model*, see for instance [Ku88], is given here, Chapter 11 has a more extensive account.

A subsector model

The subsector model combines the activities, constraints, and objective functions of a group of farms. The main activity of a farm is growing different crops and perhaps keeping livestock. Typical constraints restrict the availability of land and labor. Agricultural models become multi-period models as different growing seasons are incorporated.

Similar farms

The simplest type of agricultural subsector model is one comprised of a number of similar farms which compete for a common resource or share a common market. By "similar" is meant that they share the same technology in terms of crop possibilities, labor requirements, yields, prices, etc. Such subsector models can be used to verify if common resources are really a limiting factor at a regional level. Hired labor, for instance, might well be a constraint at the regional level.

Farm types

A different situation arises when there are fundamental differences between some of the farms in the subsector to be modeled. One common difference is in size, which means that resource availabilities are no longer similar. In the following example, it is assumed that the types of farms are distinguished mostly by size.

An example

Maximize:
regional income = income aggregated over different farm types,
Subject to:
- *for each type of farm, for each time period: a restriction on the availability of resources (land, labor, water) needed for cropping activities, and*
- *for each time period: a regional labor restriction on the availability of hired workers (temporary plus permanent).*

Other applications

Other farming applications include blending models for blending cattle feed or fertilizers at minimum cost. Distribution problems may arise from the distribution of crops or milk. Large models can be used for the management of water for irrigation, or to examine the role of livestock in an economy. Furthermore, these principles can be applied with equal success to other economic subsectors.

1.6 Summary

This chapter discussed the basic concepts of modeling. Some of the main questions addressed in this chapter are: *What is a mathematic model*, *why use mathematical models*, and *what is the role of mathematics during the construction of models*. The modeling process has been introduced as an iterative process with the steps to be executed described in detail. The latter portion of this chapter introduced several application areas in which optimization models can be used.

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