AIMMS Modeling Guide - Power System Expansion Problem

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Chapter 16

A Power System Expansion Problem

This chapter

In this chapter you will encounter a simplified power system expansion problem with uncertain electricity demand covering a single time period. Through this example you will be introduced to some methods for handling uncertainty in input data. The first approach to handle uncertainty starts with a deterministic model, and examines its sensitivity to changes in the values of uncertain parameters. This is referred to as *what-if analysis*, and is essentially a manual technique to deal with uncertainty. The second approach in this chapter goes one step further, and captures the input data associated with an entire what-if analysis into a single model formulation. This second approach is described in detail, and will be referred to as *stochastic programming*.

References

There is a vast literature on stochastic programming, but most of it is only accessible to mathematically skilled readers. The example in this chapter captures the essence of stochastic programming, and has been adapted from Malcolm and Zenios [Ma92]. Two selected book references on stochastic programming are [In94] and [Ka94].

Keywords

Linear Program, Stochastic Program, Two-Stage, Control-State Variables, What-If Analysis, Worked Example.

16.1 Introduction to methods for uncertainty

Deterministic versus stochastic models

The mathematical programming models discussed so far have had a common assumption that all the input information is known with certainty. This is known as "decision making under certainty." The main problem was to determine which decision, from a large number of candidates, yields the best result, according to some criterium. Models that account for uncertainty are known as *stochastic* models—as opposed to *deterministic* models, which do not.

Event parameters

Stochastic models contain so-called *event parameters* which do not have a priori fixed values. An event is an act of nature which falls outside the control of a decision maker. A typical example of an event parameter is the demand of products inside a decision model that determines production levels. The exact demand values are not known when the production decision has to be made, but only become known afterwards. Beforehand, it only makes sense to consider various data realizations of the event parameter demand, and somehow take them into account when making the production decision.

Assume few scenarios

The number of possible data realizations of event parameters is usually very large. In theory, it is customary to relate the event parameters to continuous random variables with infinitely many outcomes. In practical applications, however, it turns out that it is better from a computational point of view to assume a finite number of data realizations. One complete set of fixed values for all of the event parameters in a stochastic model is referred to as a *scenario*. Throughout the sequel, the assumption is made that there are only a manageable number of scenarios to be considered when solving a stochastic model. In addition, it is assumed that the likelihood of each scenario occurring is known in advance.

Control and state variables

A stochastic model contains two types of variables, namely control and state variables. *Control variables* refer to all those decision variables that must be decided at the beginning of a period prior to the outcome of the uncertain event parameters. *State variables*, on the other hand, refer to all those decision variables that are to be determined at the end of a period after the outcome of the uncertain event parameters are known. Thus, the variables representing production decisions are control variables, while variables representing stock levels are state variables.

What-if approach

A first approach to uncertainty is to build a deterministic model and assume fixed values for the uncertain event parameters. You can then perform an extensive what-if analysis to observe changes in the control variables as the result of assuming different fixed values for the event parameters. The underlying motivation of this manual approach is to discover a single set of values for the control variables that seem to be a reasonable choice for all possible scenarios.

Stochastic programming approach

A second approach to uncertainty is to formulate an extended model in which all scenarios are incorporated explicitly. By using the scenario likelihoods, the objective function can be constructed in such a way as to minimize expected cost over all possible scenarios. The advantage of this approach is that not you but the model determines a single set of values for the control variables. This approach is referred to as stochastic programming.

Two-stage versus multi-stage

The term *two-stage* will be attached to the stochastic programming approach to indicate that decisions need to be made prior to and after the realization of the uncertain events during a single time period. In Chapter 17 the generalization is made to multiple time periods, where the term *multi-stage* reflects a sequence of two-stage decisions.

16.2 A power system expansion problem

This section

In this section you will encounter a somewhat simplified but detailed example to determine new power plant design capacities to meet an increase in electricity demand. The example forms the basis for illustrating the approaches to stochastic modeling discussed in the previous section.

Design capacity ...

The design capacity of a power plant can be defined as the maximum amount of energy per second it can produce. It is assumed that energy in the form of electricity is not stored. This implies that at any moment in time, total available supply of electricity must exceed the instantaneous electricity demand.

... must be expanded

Assume that new investments in design capacity are under consideration in order to meet an increase in electricity demand. The demand is uncertain since actual demand will be strongly influenced by actual weather and economic factors. When the new capacity is installed, it remains available for an extensive period of time. This makes the design capacity investment decision a nontrivial one. When the design capacity exceeds demand, the overall capital cost is likely to be too high. Alternatively, when the capacity is insufficient, the plants can be expected to operate at peak capacity, and extra supply will need to be imported. Both these events are costly in terms of either purchase cost or operating cost.

Available plant types and no fixed sizes

In this example, electricity will be produced by three types of power plants, namely coal-fired, nuclear, and hydro plants. Each of these have their own specific capital and operating costs. In this example it is assumed that design capacity does not come in fixed sizes, but can be built in any size. Imported electricity is to be a last resource of supply, and will only be used when the installed design capacity is insufficient.

Uncertain demand

Electricity demand varies over days, weeks, seasons, and years. Rather than model demand over extended periods of time, a conscious choice has been made to only consider a single time period of one day. This choice simplifies the model to be built, but still allows for demand scenarios that typify demand throughout a planning horizon of years. The number of scenarios to be considered is assumed to be finite. Their number does not need to be large, as long as there are enough scenarios to form a representative sample of future demand.

Scenario selection

Determining a representative sample of daily future demand instances is nontrivial. When are scenarios sufficiently distinct? Can you keep their number under control? What is the effect on model solutions when particular scenarios are left out? How likely is each scenario? These questions need to be dealt with in practical applications, but are not treated in this chapter. Instead, a limited number of scenarios and their probabilities are provided without additional motivation.

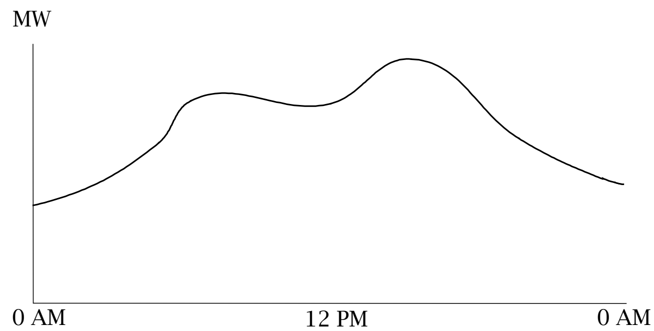

Figure 16.1: Daily load duration curve

Daily demand

The daily electricity demand is by no means constant. The demand curve is tightly linked to economic and household activities: more electricity is used at noon than at midnight. A *load duration curve*, such as Figure 16.1, reflects the electricity demand during one day. Rather than storing a continuous curve, it is more convenient to approximate such a curve by considering fixed-length time periods consisting of say one or two hours. This is shown in Figure 16.2 where there are twelve different demands, corresponding to one of twelve time periods.

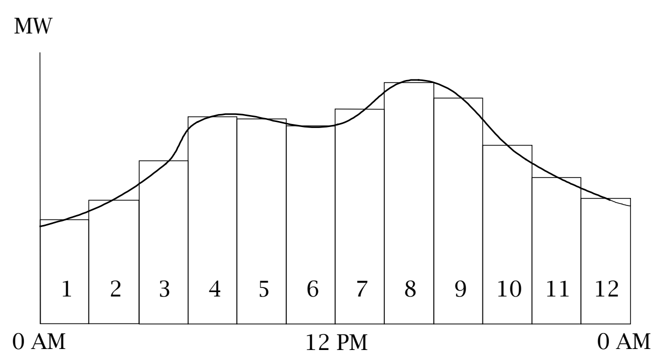

Figure 16.2: Daily load duration curve approximated by a step function

Two instead of twelve periods

Twelve time periods could be considered a large number in a study dealing with the strategic problem of capacity expansion. Lengthening the time period beyond two hours, however, would cause the approximation errors to grow. A good compromise is to first rearrange the demand periods in decreasing order of demand magnitude. In this way you obtain a *cumulative load duration curve*, as in Figure 16.3, where the slope of the curve is nearly constant. Reducing the number of time steps at this point will result in a reduced approximation error in comparison to the situation without the rearrangement (see Figure 16.4). Demand can now be realistically modeled with just two rather than twelve demand periods. In the sequel, the two corresponding demand categories will be referred to as *base load* and *peak load*.

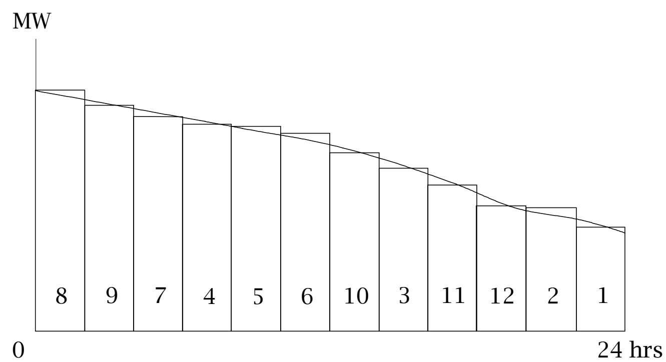

Figure 16.3: Cumulative load duration curve

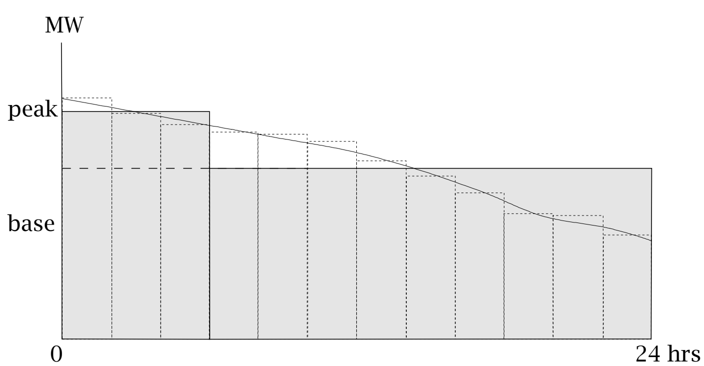

Figure 16.4: Approximating cumulative load duration curve

16.3 A deterministic expansion model

This section

In this section a deterministic version of the power system expansion problem is presented. This model forms the basis for subsequent sections that discuss stochastic approaches to deal with the uncertain demand.

Decisions to be made

The main decision to be made is the new design capacity (in [GW]) of each type of power plant. It is assumed that all levels of design capacity can be built, and that there are no decisions to be made about either the number of power plants or their size. Once capacity has been decided, it can be allocated to satisfy demand. This allocation decision can be supplemented with the decision to import electricity when available capacity is insufficient. The underlying objective is to minimize total daily cost consisting of (a fraction of) capital cost to build new design capacity, operating cost associated with the allocation decision, and the cost of importing electricity.

Deterministic daily demand requirements

In the previous section the distinction has been made between base demand and peak demand. These two categories represent instantaneous demand in GW. The duration of base demand is 24 hours, while the duration of peak demand is 6 hours. On the basis of instantaneous demand and its duration you can compute the electricity requirement per category in GWh as follows. The base electricity demand is equal to base demand multiplied by 24 [h]. The peak electricity demand is the difference between peak demand and base demand multiplied by the proper duration expressed in hours. The formula is provided at the end of the 'Notation' paragraph stated below.

Restricting nuclear usage

Due to physical plant restrictions, nuclear power can only be used to satisfy base demand requirements. In AIMMS such a restriction can be enforced by using domain restrictions on the allocation variables introduced in the model below.

Verbal model statement

The objective and the constraints that make up the simplified deterministic power expansion model are first summarized in the following qualitative model formulation.

Minimize: *total capital, import and operating costs,*

Subject to:

- *for all plant types: allocated capacity must be less than or equal to existing design capacity plus new design capacity, and*
- *for all modes: utilized capacity plus import must equal electricity demand.*

Notation

The following symbols will be used.

Indices:

p	*plant types*
k	*demand categories*

Parameters:

e_p	*existing capacity of plant type p [GW]*
cc_p	*daily fraction of capital cost of plant p* [10^3 *\$/GW]*
oc_p	*daily operating cost of plant p* [10^3 *\$/GWh]*
ic_k	*electricity import cost for category k*[10^3 *\$/GWh]*
d_k	*instantaneous electricity demand for category k [GW]*
du_k	*duration of demand for category k [h]*
r_k	*required electricity for category k [GWh]*

Variables:

x_p	*new design capacity of plant type p [GW]*
y_{pk}	*allocation of capacity to demand [GW]*
z_k	*import of electricity for category k [GWh]*

where the parameter r_k is defined as $r_k = (d_k - d_{k-1})du_k$.

Mathematical model statement

The mathematical description of the model can be stated as follows.

Minimize:

$$\sum_p cc_p(e_p + x_p) + \sum_k \left(ic_k z_k + du_k \sum_p oc_p y_{pk} \right)$$

Subject to:

$$\begin{aligned}
\sum_k y_{pk} &\leq e_p + x_p & \forall p \\
z_k + du_k \sum_p y_{pk} &= r_k & \forall k \\
x_p &\geq 0 & \forall p \\
y_{pk} &\geq 0 & \forall (p,k) \\
y_{\text{nuclear, peak}} &= 0 & \\
z_k &\geq 0 &
\end{aligned}$$

16.4 What-if approach

This section

This section presents a first approach to deal with the uncertainty in electricity demand. In essence, you consider a set of demand figures, called *scenarios* and observe the proposed design capacity for each scenario. The underlying motivation is to discover manually, through trial and error, those design capacity values that seem to be a good choice when taking into account all possible scenarios.

Model data

Consider a set of four scenarios reflecting different weather and economic conditions. Estimated figures for both base and peak load per scenario are presented in Table 16.1.

Demand [GW]	Scenario 1	Scenario 2	Scenario 3	Scenario 4
base load	8.25	10.00	7.50	9.00
peak load	10.75	12.00	10.00	10.50

Table 16.1: Estimated demand figures for different scenarios

Initial existing capacity, together with capital and operating cost figures for each plant type are summarized in Table 16.2. In addition, the cost of importing electricity is set at 200 [10^3 \$/GWh] for each demand category, and, as stated previously, the duration of the base period is 24 hours while the duration of the peak period is 6 hours.

Plant Type p	e_p [GW]	cc_p [10^3 \$/GW]	oc_p [10^3 \$/GWh]
coal	1.75	200	30.0
hydro	2.00	500	10.0
nuclear		300	20.0

Table 16.2: Existing capacity and cost figures per plant type

Constructing plans from scenarios

A set of design capacities will be referred to as a *plan*. A plan can be determined by computing the optimal design capacities for a particular demand scenario. By repeating the computation for each of the four demand scenarios, the four plans in Table 16.3 are obtained. Note that coal and hydro power plants seem to be the most attractive options in all four plans. No nuclear capacity is installed.

Selecting the cheapest plan

Eventually, only a single plan can be implemented in reality. An obvious first choice seems to be the cheapest plan from the table above, namely, plan III. However, what happens when plan III is implemented and a scenario other than scenario 3 occurs? The answer to this question is illustrated in Table 16.4. You can see that total cost increases dramatically for the other scenarios. This increase is due to the relatively expensive cost to import electricity to meet the increased demand. If all scenarios are equally likely, then the average cost is 8,116.25 [10^3 \$].

Plan	Based on	Total Capacity [GW]			Total
		coal	hydro	nuclear	Cost [10^3 $]
I	scenario 1	2.50	8.25		7055.0
II	scenario 2	2.00	10.00		8160.0
III	scenario 3	2.50	7.50		6500.0
IV	scenario 4	1.75	8.75		7275.0

Table 16.3: Optimal design capacities for individual scenarios only

Comparing all plans

As indicated in the previous paragraph, the cheapest plan may not be the best plan. That is why it is worthwhile to look at all other plans, to see how they perform under the other scenarios. An overview is provided in Table 16.5. In this comparison, plan III turns out to be the worst in terms of average cost, while plan I scores the best.

Is there a better plan?

Each plan was produced on the basis of one of the selected demand scenarios. You can always dream up another (artificial) scenario just to produce yet another plan. Such a plan could then be evaluated in terms of the average cost over the four scenarios, and perhaps turn out to be a better plan. For instance, an obvious choice of artificial demand figures is the average demand over the four scenarios. Solving the model for these demand figures results in a plan with an average cost of 7685.35 [10^3 $], slightly better than plan I. Even though what-if analysis has provided some insight into the selection of a better plan, there is a clear need for an approach that provides the best plan without having to search over artificial scenarios. Such an approach is explained in the next section.

16.5 Stochastic programming approach

This section

This section presents a second and more sophisticated approach to deal with uncertainty in electricity demand when compared to the what-if approach of the previous section. Essentially, all demand scenarios are included in the model simultaneously, and their average cost is minimized. In this way, the model solution presents those particular design capacity values that seem to be a good choice in the light of all scenarios weighted by their probability. The model results of the worked example are compared to the solutions presented in the previous section.

Two-stage model

In the mathematical model of this chapter, the electricity demand covers a single time period. Prior to this period, the exact demand values are not known, but a capacity design decision must be made. This is referred to as the *first stage*. As the realization of electricity demand becomes known, the allocation of the already decided design capacity is made, together with the decision of

	Capacity Imported [GW]	**Total Cost [10^3 $]**
scenario 1	0.75	7805.0
scenario 2	2.00	10250.0
scenario 3		6500.0
scenario 4	0.50	7910.0

Table 16.4: The consequences of plan III for all scenarios

how much to import. This is the *second stage*. Using the terminology introduced in Section 16.1, the capacity design decision variables form the *control variables*, while the allocation and import decision variables form the *state variables*.

Expected cost

Both the allocation and import decision variables form the state variables, which will assume separate values for each scenario. This implies that there are also separate values associated with total operating cost and total import cost. It does not make sense to add all these total cost values in the cost minimization objective function. A weighted average over scenarios, however, seems a better choice, because the objective function then reflects the expected daily cost of the entire power supply system over a planning period. The weights are then the probabilities associated with the scenarios.

Notation

The following symbols will be used. Note that most of them are similar to the ones already presented in the previous section, except that a scenario index has been added. As to be expected, the first stage (control) variable x does not have a scenario index, while all second stage (state) variables are naturally linked to a scenario.

Indices:

p	*plant types*
k	*demand categories*
s	*scenarios*

Parameters:

e_p	*existing capacity of plant type p [GW]*
cc_p	*annualized capital cost of plant p [10^3 $/GW]*
oc_p	*operation cost of plant p [10^3 $/GWh]*
ic_k	*electricity import cost for category k[10^3 $/GWh]*
d_{ks}	*instantaneous electricity demand for k and s [GW]*
du_{ks}	*duration of demand for category k and scenario s [h]*
r_{ks}	*required electricity for k and s [GWh]*
pr_s	*probability of scenario s [-]*

Plan	Total Cost [10^3 \$] per Scenario				Expected Cost [10^3 \$]
	1	2	3	4	
I	**7055.0**	9500.0	6785.0	7415.0	7688.8
II	7620.0	**8160.0**	7350.0	7710.0	7710.0
III	7805.0	10250.0	**6500.0**	7910.0	8116.3
IV	7350.0	9615.0	6825.0	**7275.0**	7766.3

Table 16.5: Plans compared in terms of cost

Variables:

x_p	*new design capacity of plant type p [GW]*
y_{pks}	*allocation of capacity to each demand realization [GW]*
z_{ks}	*electricity import for scenario s and category k [GW]*
v_s	*total import and operating cost for scenario s [10^3 \$]*

where the parameter r_{ks} is defined as $r_{ks} = (d_{ks} - d_{k-1,s})du_{ks}$.

Mathematical model statement

The mathematical description of the stochastic model below resembles the model description in the previous section. The main difference is the formulation of the objective function. The capital cost determination associated with existing and new design capacity remains unchanged. All other cost components are scenario-dependent, and a separate definition variable v_s is introduced to express the expected operating and importing cost in a convenient manner.

Minimize:

$$\sum_p cc_p(e_p + x_p) + \sum_s pr_s v_s$$

Subject to:

$$\sum_k y_{pks} \le e_p + x_p \qquad \forall (p,s)$$

$$z_{ks} + du_{ks}\sum_p y_{pks} = r_{ks} \qquad \forall (k,s)$$

$$\sum_k \left(ic_k z_{ks} + du_{ks} \sum_p oc_p y_{pks} \right) = v_s \qquad \forall s$$

$$x_p \ge 0 \qquad \forall p$$

$$y_{pks} \ge 0 \qquad \forall (p,k,s)$$

$$y_{\text{nuclear, peak}, s} = 0 \qquad \forall s$$

$$z_{ks} \ge 0 \qquad \forall s$$

$$v_s \ge 0 \qquad \forall s$$

Plan	**Total Capacity [GW]**			**Expected**
	coal	**hydro**	**nuclear**	**Cost [10^3 $]**
I	2.50	8.25		7688.75
II	2.00	10.00		7710.00
III	2.50	7.50		8116.25
IV	1.75	8.75		7766.25
V	3.00	8.25	0.75	7605.00

Table 16.6: Optimal design capacities and expected cost per plan

Model results

The solution of the stochastic expansion model produces an entire new plan, which will be referred to as plan V. The design capacity values (both existing and new) and the corresponding expected cost values associated with the original four plans plus plan V are listed in Table 16.6. By construction, the expected cost of plan V is the lowest of any plan.

	allocation [GW]			**import**	**surplus**	**deficit**	**total**
	coal	**hydro**	**nuclear**	[GW]	**capacity** [GW]		**costs** [10^3 $]
Scenario 1					1.25		7380.0
base		8.25					
peak	2.50						
Scenario 2							8370.0
base	1.00	8.25	0.75				
peak	2.00						
Scenario 3					2.00		7110.0
base		7.50					
peak	1.75	0.75					
Scenario 4					1.50		7560.0
base		8.25	0.75				
peak	1.50						

Table 16.7: Optimal allocations for individual scenarios

Nuclear power plant selected

In Plan V nuclear power is selected as a source of electricity. The reason is that nuclear power plants have lower capital costs than hydro-electric power plants. This fact helps to keep the costs down in scenarios other than the most restrictive one, namely Scenario 2. The optimal allocations corresponding to Plan V are given in Table 16.7.

16.6 Summary

In this chapter, two methods for dealing with uncertainty were illustrated using a power plant capacity expansion example. Both methods were based on the use of scenarios designed to capture the uncertainty in data. The first method is referred to as *what-if analysis*, and essentially computes the consequences for each individual scenario by solving a sequence of deterministic models. The second method is referred to as *stochastic programming*, and considers all scenarios simultaneously while minimizing expected cost. The resulting model increases in size in proportion to the number of scenarios. Based on the example, the solution of the stochastic programming formulation was shown to be superior to any of the solutions derived from the manual what-if analysis.

Exercises

16.1 Implement the deterministic formulation of the Power System Expansion model described in Section 16.3, and perform the what-if experiments described in Section 16.4 using Aimms.

16.2 Implement the stochastic formulation of the Power System Expansion model described in Section 16.5, and compare the optimal solution with the solutions of the what-if experiments.

16.3 Set up an experiment in Aimms to investigate the sensitivity of the optimal stochastic programming solution to changes in the initial equal scenario probabilities of .25.

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