AIMMS Modeling Guide - Farm Planning Problem

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Chapter 11

A Farm Planning Problem

This chapter

In this chapter you will find a worked example of a simplified farm planning problem in a developing country. Every year a farmer must decide what crops to grow, thereby taking into account such limiting resources as land, labor and water. His objective is to maximize farm revenue. This simplified farm planning problem can be translated into a linear optimization model. Such a model is typically found as a submodel in larger models which focus on agricultural questions concerning an entire region. Even though the model developed here is relatively small in terms of symbolic constraints, the number of identifiers is relatively large.

References

The material of this chapter has been adapted from "Modeling for Agricultural Policy and Project Analysis" by G.P. Kutcher, A. Meeraus, and G.T. O'Mara [Ku88]. A text book on agricultural modeling is [Ha86].

Keywords

Linear Program, Measurement Units, Sensitivity Analysis, What-If Analysis, Worked Example.

11.1 Problem description

Three main inputs

The three main inputs to agricultural production are *land*, *labor* and *water*. Their requirements vary throughout the year for different crop activities. In this chapter all requirements will be specified on a monthly basis, and are assumed to be known with certainty.

Land

Crop rotation is the practice of growing different crops, in sequence, on the same land. The specification of all relevant crop rotations and time shifts in real-world applications is a non-trivial task, and requires the aid of an agronomist. In this chapter, a monthly calendar is maintained. The periods in which crops can be grown, are given in Table 11.1. An entry in this table denotes the fraction of a month that land will be occupied by a particular crop. The total amount of land available is assumed to be 10 hectares.

	wheat	**beans**	**onions**	**cotton**	**maize**	**tomatoes**
Jan	1	1	1			
Feb	1	1	1			
Mar	1	1	1	.5		
Apr	1	1	1	1		
May	1		.25	1	.25	
Jun				1	1	
Jul				1	1	.75
Aug				1	1	1
Sep				1	1	1
Oct				1	.5	1
Nov	.5	.25	.5	.75		.75
Dec	1	1	1			

Table 11.1: Land occupation [-]

Labor requirements

Labor is one of the major requirements for crop activities, and is supplied by the farmer's family as well as outside workers. The use of labor is disaggregated over the year, because labor is employed for different activities (land preparation, planting, maintenance and harvesting) requiring different intensities. For each crop pattern there will be a set of labor requirements that vary each month. Table 11.2 gives the labor requirements in the same pattern as land use seen in Table 11.1. Each number in the table represents the amount of labor [hr] required that month for growing one hectare of the corresponding crop.

	wheat	**beans**	**onions**	**cotton**	**maize**	**tomatoes**	**hours**
Jan	14	6	41				160
Feb	4	6	40				160
Mar	8	6	40	40			184
Apr	8	128	155	40			176
May	137		19	72	34		168
Jun				16	40		176
Jul				12	57	136	176
Aug				16	64	120	176
Sep				8	35	96	176
Oct				46	9	56	168
Nov	19	60	89	34		48	176
Dec	11	6	37				176

Table 11.2: Crop labor requirements [hr/ha] and monthly hours [hr/man]

Workers

In addition to the already available family labor, outside workers can be hired as permanent workers or temporary workers. Permanent family labor consists of 1.5 person, and costs $ 4,144 per man for an entire year. Permanent outside labor costs $ 5,180 per man for a year, while temporary labor costs is $ 4 per hour. The number of working hours differs per month and is listed in the last column of Table 11.2. Note that the fractional value of 1.5 for family labor can be viewed as an average over several similar farms or as an indication of parttime labor activity.

Water

Water is another major requirement for agricultural production, and can be supplied from surface water distribution or from groundwater. In this chapter it is assumed that the total amount of water available to the farmer each month is restricted to 5 kcub. Furthermore, there is an overall annual limit on the use of water equal to 50 kcub. The price of water is fixed for the entire year, and amounts to $ 10 per kcub. The crop water requirements for the farm are given in Table 11.3.

	wheat	**beans**	**onions**	**cotton**	**maize**	**tomatoes**
Jan	0.535	0.438	0.452			
Feb	0.802	0.479	0.507			
Mar	0.556	0.505	0.640	0.197		
Apr	0.059	0.142	0.453	0.494		
May				1.047	0.303	
Jun				1.064	0.896	
Jul				1.236	1.318	0.120
Aug				0.722	0.953	0.241
Sep				0.089	0.205	0.525
Oct						0.881
Nov	0.373	0.272				0.865
Dec	0.456	0.335	0.305			

Table 11.3: Crop water requirements [kcub/ha])

Family consumption

Most farms in developing countries place an emphasis on the production of their own food, since it makes them self-sufficient. Rather than fixing certain crops for family consumption, a slightly more general treatment is to allow a choice from a set of a priori constructed consumption bundles. Each such bundle is composed of different crops, and is by itself sufficient to feed the family. A menu of balanced meals can be derived from it. By considering a combination of alternative bundles, a farmer can take his overall revenue into account while deciding on his own consumption.

Consumption data

Consider three alternative consumption bundles (in tons per year) that the farmer can choose from. The first bundle contains 1.20 tons of beans, 0.15 tons of maize, and 0.25 tons of tomatoes. The second bundle contains 0.73 tons of beans, 1.50 tons of maize, and 0.25 tons of tomatoes. The third bundle contains 0.70 tons of beans, 1.00 ton of maize, and 0.75 tons of tomatoes. It is assumed that any combination of these three bundles may be selected by the farmer.

Yields and revenues

Based on previous years the expected yield for each crop can be estimated in advance. For instance, the *yield* of growing cotton on one hectare of land will be 1.5 tons of cotton. Similar figures exist for the yield of the other crops. Furthermore, the *price* of crops determines the farm revenue, and price indications are also assumed to be known. In table 11.4 the relevant yields and prices are presented.

	yield [ton/ha]	**price [$/ton]**
wheat	1.50	1000
beans	1.00	2000
onions	6.00	750
cotton	1.50	3500
maize	1.75	700
tomatoes	6.00	800

Table 11.4: Crop yields and prices

11.2 Model formulation

This section

In this section the above description is translated into an optimization model. First, an informal verbal presentation of the model is provided, followed by the extensive notation needed to describe all aspects of the model. The objective function and each constraint is developed separately. A model summary is listed at the end.

Verbal model statement

By considering the basic choices of the farmer and his limited resources, it is fairly straightforward to describe his objective and constraints in a compact verbal manner.

Maximize: *total net farm revenue which is the revenue from sales minus the costs associated with labor and water.*

Subject to:

- *for all months: the land used for cropping activities must be less than or equal to the land available,*
- *for all months: the labor needed for cropping activities must be less than or equal to available family labor plus hired permanent labor plus hired temporary labor,*
- *for all months: water needed for cropping activities must be less than or equal to the monthly water limit,*

- *the annual amount of water needed must be less than or equal to the annual water limit, and*
- *for all crops: the amount produced must be equal to the amount to be consumed plus the amount to be sold.*

Notation

The following notation is based as much as possible on the use of a single letter for each identifier for reasons of compactness. Such compact notation is not recommended for practical models built with a system such as AIMMS, because short names do not contribute to the readability and maintainability of computerized models.

Indices:

c	*crops*
t	*months*
b	*consumption bundles*

Parameters (crop related):

y_c	*yield of crop c [ton/ha]*
p_c	*price of crop c [$/ton]*
d_{cb}	*amount of crop c in bundle b [ton]*

Parameters (land related):

L	*land available [ha]*
l_{tc}	*fraction of month t that crop c occupies land [-]*

Parameters (labor related):

v_{tc}	*labor required of crop c during month t [hr/ha]*
r^F	*annual wage rate family labor [$/man]*
r^P	*annual wage rate permanent labor [$/man]*
r^T	*hourly wage rate temporary labor [$/hr]*
h_t	*working hours in month t [hr/man]*
V^F	*family labor available [man]*

Parameters (water related):

W	*annual amount of water available [kcub]*
w_t	*limit on use of water in month t [kcub]*
R_{tc}	*water requirement for crop c in month t [kcub/ha]*
p^W	*price of water [$/kcub]*

Variables:

x_c	*amount of crop c planted [ha]*
V^P	*permanent labor hired [man]*
V_t^T	*temporary labor hired in t [hr]*
s_c	*sales of crop c [ton]*
z_b	*fraction of bundle b consumed [-]*

Land limitation

Land use for crops is described in Table 11.1, in which an entry denotes the fraction of a month that land will be occupied by a particular crop. It may take you a while to get used to the definition of this table, but it represents a compact manner to describe both the timing and the use of land for the production of crops. A fraction of 1 means that land is to be occupied during the entire month for the corresponding crop. A fraction of less than 1 indicates that land is used either during the last portion of a month when the crop is sown, or during the first portion of a month when the crop is harvested and the land is cleared for another crop. With the use of Table 11.1, the resulting land limitation constraint assumes a simple format.

$$\begin{aligned} \sum_c l_{tc} x_c &\leq L \quad \forall t \quad \text{[ha]} \\ x_c &\geq 0 \end{aligned}$$

Labor requirements

Labor requirements for growing a particular crop on one hectare of land can be found in Table 11.2. In addition to the available family labor, both permanent and/or temporary labor may need to be hired. Note that permanent and temporary labor is not expressed in the same unit. That is why the conversion factor h_t [hr/man] is used to express the following constraint on labor in terms of hours.

$$\begin{aligned} \sum_c v_{tc} x_c \leq h_t(V^F + V^P) + V_t^T \qquad &\forall t \qquad \text{[hr]} \\ V^P \geq 0, \quad V_t^T \geq 0 \qquad &\forall t \end{aligned}$$

Water requirements

There is a monthly and an annual restriction on the use of water. The mathematical form of both constraints is similar to the constraints on land and labor discussed previously.

$$\begin{aligned} \sum_c R_{tc} x_c &\leq w_t \quad \forall t \quad \text{[kcub]} \\ \sum_{tc} R_{tc} x_c &\leq W \quad \quad \text{[kcub]} \end{aligned}$$

Family consumption

The amount of each crop produced during the year is meant to be sold, with the exception of those amounts that are to be used for family consumption. As indicated before, a combination of consumption bundles is to be selected to satisfy the family needs. Such a combination can be formulated as a convex combination, and is implemented by attaching a nonnegative weight to each bundle, and letting the sum of the weights be equal to one. This approach makes sure that there is enough food for the family, but allows for variation in the weights to obtain maximal farm revenue from sales.

$$\begin{aligned} y_c x_c &= \sum_b d_{cb} z_b + s_c && \forall c \quad \text{[ton]} \\ \sum_b z_b &= 1 \\ s_c &\geq 0 && \forall c \\ z_b &\geq 0 && \forall b \end{aligned}$$

Objective function

The objective is to maximize farm profit over the year, which is equal to revenue from sales minus the costs associated with labor and water. Note that in this simplified model, the cost of seedlings is ignored.

$$\sum_c p_c s_c - r^F V^F - r^P V^P - r^T \sum_t V_t^T - p^W \sum_{tc} R_{tc} x_c$$

Mathematical model summary

The mathematical description of the model can now be summarized as follows.

Maximize:

$$\sum_c p_c s_c - r^F V^F - r^P V^P - r^T \sum_t V_t^T - p^W \sum_{tc} R_{tc} x_c$$

Subject to:

$$\begin{aligned} \sum_c l_{tc} x_c &\leq L && \forall t \\ \sum_c v_{tc} x_c &\leq h_t (V^F + V^P) + V_t^T && \forall t \\ \sum_c R_{tc} x_c &\leq w_t && \forall t \\ \sum_{tc} R_{tc} x_c &\leq W \\ y_c x_c &= \sum_b d_{cb} z_b + s_c && \forall c \\ \sum_b z_b &= 1 \\ x_c &\geq 0 && \forall c \\ V^P &\geq 0 \\ V_t^T &\geq 0 && \forall t \\ s_c &\geq 0 && \forall c \\ z_b &\geq 0 && \forall b \end{aligned}$$

11.3 Model results

This section

In this section you will find a summary of the optimal solution that you yourself should observe after having implemented the model in AIMMS. In addition, there are some comments regarding experiments that you could carry out to investigate the effect of parameter changes on the value of the optimal solution.

Optimal solution

The optimal cropping pattern and the use of crop returns are summarized in Table 11.5. As expected, the crops that are not sold, are used for family consumption. The optimal combination of bundles was found to be bundle 1 (7%) and bundle 3 (93%). Note that beans and maize are only grown to meet family consumption. The optimal yearly net revenue turns out to be $ 49,950.

	crop x_c [ha]	**yield** $y_c x_c$ [ton]	**sales** s_c [ton]	**consumption** $\sum_b d_{cb} z_b$ [ton]
wheat				
beans	0.74	0.74		0.74
onions	6.33	37.96	37.96	
cotton	2.94	4.40	4.40	
maize	0.54	0.94		0.94
tomatoes	5.55	33.29	32.58	0.71

Table 11.5: Optimal crop-related solution

Use of labor

Besides the fixed amount of permanent family labor of 1.5 man, the optimal solution also indicates a need for permanent hired labor of 0.54 man. As stated before, these fractional numbers could be interpreted as an average over several similar farms, or as indication of permanent parttime work. The optimal amount of temporary labor hired is expressed in Table 11.6.

	V_t^T [hr]		V_t^T [hr]
Jan		Jul	461.72
Feb		Aug	388.43
Mar		Sep	216.25
Apr	833.86	Oct	108.24
May	7.45	Nov	614.86
Jun		Dec	

Table 11.6: Optimal temporary labor hired

Shadow price of labor

In AIMMS you can turn on the property `ShadowPrice` for each individual symbolic constraint. If you do so for the labor requirement constraint, you will observe that the shadow prices for January, February, June and December are all zero. This indicates an overcapacity of permanent labor during these months. In March the amount of available permanent labor is exactly enough to meet labor requirements. In all other months, temporary labor is hired to meet labor requirements.

Use of water

The water requirement constraints are only binding during the months March, July and November. The corresponding shadow prices for these constraints are nonzero and reflect the binding use of water. The annual use of water amounts to 47.32 kcub, which is less than the total annual limit.

Changing the water limit

If the annual supply of water is not entirely known a priori, the farmer may want to look how the optimal solution changes as a result of different values. You can implement such an experiment in AIMMS by creating a parametric curve with annual water supply along the x-axis and crop assignments along the y-axis. Figure 11.1 represents such a curve with the annual water limit between 40 and 50 kcub. The observed sensitivity is not so strong, so that the farmer can make a decision even if the annual water limit varies within the above range.

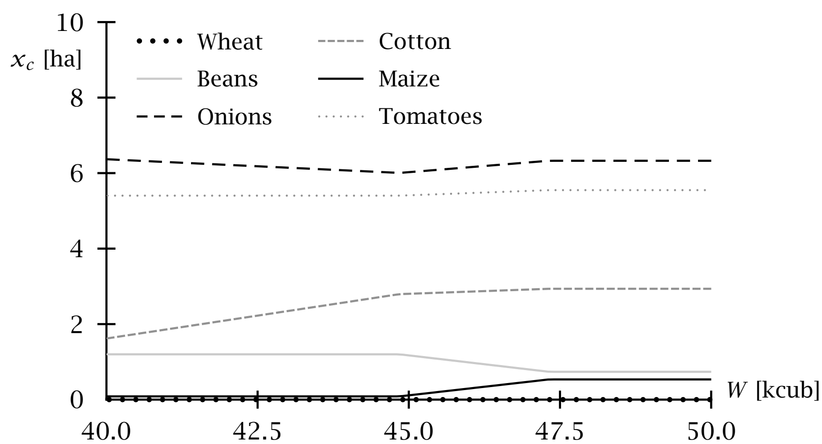

Figure 11.1: Optimal crops as function of annual water limit

Changing the land availability

A similar experiment can be made when the farmer has the opportunity to use additional land. The parametric curve in Figure 11.2 shows the sensitivity of optimal cropping patterns with respect to land. As more land becomes available, the amount of cotton increases while the amount of onions and maize decreases. You may verify for yourself that total net revenue increases by more than 20 % as the amount of available land doubles.

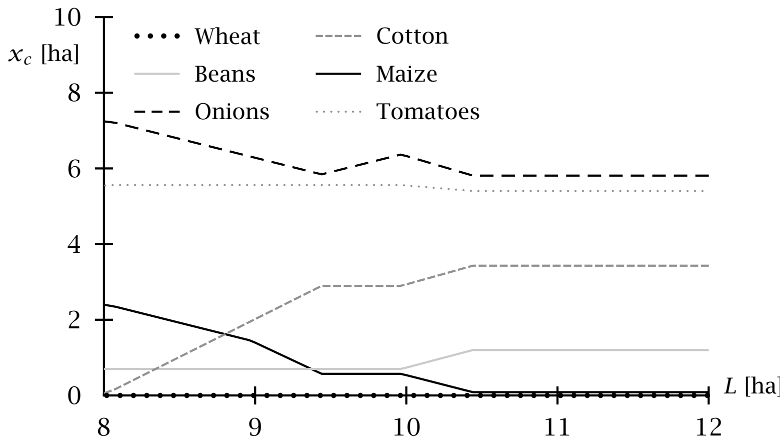

Figure 11.2: Optimal crops as function of available land

11.4 Summary

In this chapter a simplified farming problem was introduced together with a small data set for computational purposes. The corresponding model turned out to be a straightforward linear program with constraints on land, labor, water and family consumption. What-if experiments were performed to determine the sensitivity of selected parameters on the optimal solution.

Exercises

11.1 Implement the mathematical program described at the end of Section 11.2 using the example data provided in Section 11.1. Verify that the optimal solution produced with AIMMS is the same as the solution provided in Tables 11.5 and 11.6.

11.2 Add all quantity and unit information to your model in AIMMS, and check whether the units are consistent throughout the model.

11.3 Use the parametric curve object in AIMMS to reproduce the sensitivity experiments described in Section 11.3.

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