AIMMS Modeling Guide - Employee Training Problem

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AIMMS B.V.
Diakenhuisweg 29-35
2033 AP Haarlem
The Netherlands
Tel.: +31 23 5511512

AIMMS Inc.
11711 SE 8th Street
Suite 303
Bellevue, WA 98005
USA
Tel.: +1 425 458 4024

AIMMS Pte. Ltd.
55 Market Street #10-00
Singapore 048941
Tel.: +65 6521 2827

AIMMS
SOHO Fuxing Plaza No.388
Building D-71, Level 3
Madang Road, Huangpu District
Shanghai 200025
China
Tel.: ++86 21 5309 8733

Email: info@aimms.com
WWW: www.aimms.com

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Part III

Basic Optimization Modeling Applications

Chapter 8

An Employee Training Problem

This chapter

This chapter introduces a personnel planning problem and its corresponding multi-period model. The model includes a (stock) balance constraint which is typical in multi-period models involving state and control type decision variables. A time lag notation is introduced for the backward referencing of time periods. Initially, a simplified formulation of the model is solved and it is shown that rounding a fractional linear programming solution can be a good alternative to using an *integer* programming solver. Finally, the full model complete with random variables is considered, and an approach based on the use of probabilistic constraints is presented.

References

Problems of this type can be found in, for instance, [Wa75], [Ep87], and [Ch83].

Keywords

Linear Program, Integer Program, Control-State Variables, Rounding Heuristic, Probabilistic Constraint, Worked Example.

8.1 Hiring and training of flight attendants

Hiring and training

Consider the following personnel planning problem. The personnel manager of an airline company must decide how many new flight attendants to hire and train over the next six months. Employment contracts start at the beginning of each month. Due to regulations each flight attendant can only work up to 150 hours a month. Trainees require two months of training before they can be put on a flight as regular flight attendants. During this time a trainee is effectively available for 25 hours a month. Due to limited training capacity, the maximum number of new trainees each month is 5.

Job notification

Throughout the year, flight attendants quit their jobs for a variety of reasons. When they do, they need to notify the personnel manager one month in advance. The uncertainty in the number of flight attendants leaving the company in future months, makes it difficult for the personnel manager to make long term plans. In the first part of this chapter, it is assumed that all the leave notifications for the next six months are known. This assumption is relaxed at the end of the chapter.

Example data

The airline problem is studied using the following data. At the beginning of December, 60 flight attendants are available for work and no resignations were received. Two trainees were hired at the beginning of November and none in December. The cost of a flight attendant is $5100 a month while the cost of a trainee is $3600 a month. The flight attendant requirements in terms of flight hours are given in Table 8.1 for the months January to June. The table also includes the (known) number of flight attendants who will hand in their resignation, taking effect one month thereafter.

months	**required hours**	**resignations**
January	8,000	2
February	9,000	
March	9,800	2
April	9,900	
May	10,050	1
June	10,500	

Table 8.1: Flight attendant requirements and resignations

8.2 Model formulation

Control and state variables

The main decision to be made by the personnel manager is the number of trainees to be hired at the beginning of each month. Prior to this decision, both the number of trainees and the number of flight attendants are unknown, and these represent two types of decision variables. An important observation is that once the decision regarding the trainees has been made then the number of available flight attendants is determined. This leads to the distinction between *control variables* and *state variables*. Control variables are independent decision variables while state variables are directly dependent on the control variables. In mathematical models both are considered as decision variables.

Balance constraints

The distinction between control and state variables often leads to the use of so-called *balance constraints*. These are often equalities defining the state variable in terms of the control variables. Such constraints typically occur in multi-period models in which state variables such as stocks of some sort are recorded over time.

Verbal model statement

A verbal model statement of the problem is based on the notion of a balance constraint for the number of attendants plus a constraint reflecting the personnel requirement in terms of flight hours.

Minimize: *total personnel costs,*
Subject to:

- *for all months: the number of flight attendants equals the number of flight attendants of the previous month minus the number of flight attendants that resigned the previous month plus the number of trainees that have become flight attendants,*
- *for all months: the number of attendant flight hours plus the number of trainee hours must be greater than or equal to the monthly required flight attendant hours.*

Notation

The verbal model statement of the personnel planning problem can be specified as a mathematic model using the following notation.

Index:

t	*time periods (months) in the planning interval*

Parameters:

c	*monthly cost of one flight attendant*
d	*monthly cost of one trainee*
u	*monthly number of hours of one flight attendant*
v	*monthly number of hours of one trainee*
m	*maximum number of new trainees each month*
r_t	*required flight attendant hours in t*
l_t	*number of flight attendants resigning in t*

Variables:

x_t	*number of flight attendants available in t*
y_t	*number of trainees hired in t*

Time specification

It is important to explicitly specify the precise time that you assume the decisions take place. It makes a difference to your model constraints whether something takes place at the beginning or at the end of a period. A good rule is to be consistent for all parameters and variables in your model. Throughout this section it is assumed that all events take place at the beginning of each monthly period.

Attendant balance

The flight attendant balance constraint is a straightforward book keeping identity describing how the number of flight attendants varies over time.

$$x_t = x_{t-1} - l_{t-1} + y_{t-2} \qquad \forall t$$

Lag and lead notation . . .

The index t refers to periods in the planning period. The time-lag references $t-1$ and $t-2$ refer to one and two periods prior to the current period t, respectively. When the current period happens to be the first period, then the references x_{t-1} and y_{t-2} refer to the past. They do not represent decision variables but instead they define the initial conditions of the problem. When using the balance equation, care must be taken to ensure that it is only used for time periods within the planning interval. In this chapter the planning interval is specified by the months January through June.

... in AIMMS

In AIMMS there is a special type of set called *horizon*, which is especially designed for time periods. In each horizon, there is a *past*, a *planning interval* and a *beyond*. Any variable that refers to a time period outside the planning interval (i.e. past and beyond) is automatically assumed to be data (fixed). This feature simplifies the representation of balance constraints in AIMMS.

Requirement constraint

The personnel requirement constraint is not stated in terms of persons but in term of hours. This requires the translation from persons to hours.

$$ux_t + v(y_t + y_{t-1}) \geq r_t \qquad \forall t$$

Note that the trainees who started their training at the beginning of the current month or the previous month only contribute $v < u$ hours to the total availability of flight attendant hours.

Objective function

The objective function is to minimize total personnel cost.

Minimize:

$$\sum_t cx_t + d(y_t + y_{t-1})$$

Model summary

The following mathematical statement summarizes the model.

Minimize:

$$\sum_t cx_t + d(y_t + y_{t-1})$$

Subject to:

$$\begin{aligned} x_t &= x_{t-1} - l_{t-1} + y_{t-2} && \forall t \\ ux_t + v(y_t + y_{t-1}) &\geq r_t && \forall t \\ x_t &\geq 0, \quad \textit{integer} && \forall t \\ 0 \leq y_t &\leq m, \quad \textit{integer} && \forall t \end{aligned}$$

8.3 Solutions from conventional solvers

Integer requirement

The above model is an integer programming model since all decision variables must assume integer values. You may relax this requirement for the variable x_t (the number of flight attendants) because in the balance constraint defining x_t in terms of y_t (the number of new trainees), this value of x_t is automatically integer when y_t is integer.

Optimal integer solution

The model, when instantiated with the small data set, can easily be solved with any integer programming solver. The solution found by such a solver is displayed in Table 8.2 and its corresponding optimal objective function value is $2,077,500.

	flight attendants	**trainees**
November		2
December	60	
January	62	5
February	60	2
March	65	2
April	65	4
May	67	
June	70	

Table 8.2: The optimal integer solution

Suboptimal integer solution

As the sizes of the underlying data sets are increased, it may become impractical to find an optimal integer solution using a conventional integer programming solver. Under these conditions, it is not atypical that the conventional solver finds one or more suboptimal solutions in a reasonable amount of computing time, but subsequently spends a lot of time trying to find a solution which is better than the currently best solution. In practical applications, this last best solution may very well be good enough. One way to obtain such suboptimal solutions is to specify optimality tolerances.

Setting optimality criteria

Whenever you are solving large-scale integer programming models, you are advised to use solution tolerance settings in an effort to avoid long computational times. In AIMMS you can specify both a *relative optimality tolerance* and an *absolute optimality tolerance*. The relative optimality tolerance `MIP_relative_optimality_tolerance` is a fraction indicating to the solver that it should stop as soon as an integer solution within 100 times `MIP_relative_optimality_tolerance` percent of the global optimum has been found. Similarly to the relative optimality tolerance, the absolute optimality tolerance `MIP_absolute_optimality_tolerance` is a number indicating that the solver should terminate as soon as an integer solution is within `MIP_absolute_optimality_tolerance` of the global optimum.

Setting priorities

Another feature available in AIMMS that can be used to reduce the solving time for large integer programs is *priority setting*. By assigning priority values to integer variables, you directly influence the order in which variables are fixed during the search by a solver. For instance, by setting low positive priority values for the y_f variables (the number of new trainees to be hired), and letting these values increase as time progresses, the branch and bound solution method will decide on the number of trainees to be hired in the same order as in the set of months. Experience has demonstrated that setting integer priorities to mimic a natural order of decisions, is likely to decrease computational time. This is particularly true when the size of the data set grows.

8.4 Solutions from rounding heuristics

Examining relaxed solution

By *relaxing* (i.e. neglecting) the integer requirement on the variables x_t and y_t, the personnel planning model becomes a linear program. In general, linear programs are easier to solve than integer programs and this is particularly true with large data instances. However, the optimal solution is not necessarily integer-valued. The question then arises whether a simple rounding of the linear programming solution leads to a good quality suboptimal integer solution. This question is investigated using the optimal linear programming solution presented in Table 8.3.

	flight attendants	**trainees**
November		2
December	60	
January	62.00	4.76
February	60.00	2.24
March	64.76	1.20
April	65.00	4.80
May	66.20	
June	70.00	

Table 8.3: The optimal LP solution

Rounding up

Rounding the values of both x_t (the flight attendants) and y_t (the trainees) in the relaxed solution violates the balance constraint. Simply rounding the y_t variables upwards and re-calculating the x_t variables does result in a feasible solution for this data set.

Rounding with reserves

A tighter variant of the heuristic of the previous paragraph is to round downwards as long as there is a reserve of at least one trainee, and to round upwards when there is no such a reserve. The solution obtained from both rounding variants are contained in Table 8.4. Note that none of the two rounded solutions are as good as the optimal integer solution. A skeleton algorithm for the second variant can be written as

```
FOR (t) DO
    IF reserve < 1
        THEN y(t) := ceil[y(t)]
        ELSE y(t) := floor[y(t)]
    ENDIF
    Update reserve
ENDFOR
```

This skeleton algorithm can form the basis for an implementation in AIMMS. Of course, the above heuristic is only one of many possible rounding procedures. A more robust heuristic should register not only the reserve created by rounding, but also the number of extra training hours required. However, this refinement is trickier than you might think at first!

	LP		Rounded up		Rounded with reserves	
	x_t	y_t	x_t	y_t	x_t	y_t
November		2		2		2
December	60		60		60	
January	62.00	4.76	62	5	62	5
February	60.00	2.24	60	3	60	3
March	64.76	1.20	65	2	65	1
April	65.00	4.80	66	5	66	5
May	66.20		68		67	
June	70.00		72		71	
Total costs	2,072,196		2,112,300		2,094,900	

Table 8.4: The rounded solutions

8.5 Introducing probabilistic constraints

Uncertain resignations

Until now, it was assumed that the number of resignations was known in advance for each month in the planning interval. Without this assumption, the number of resignations each month is not a parameter but instead a random variable with its own distribution. To obtain an insight into the distribution, it is necessary to statistically analyse the resignation data. The analysis should be based on both historic records and information about personnel volatility in the current market place.

Observed distributions

Assume that such a statistical data analysis concludes that resignation data is independently normally distributed with means and variances as presented in Table 8.5. This table also contains critical resignation levels used in the following probabilistic analysis.

	mean	**variance**	$\alpha = 0.01$	$\alpha = 0.02$	$\alpha = 0.05$
January	1.15	0.57	2.48	2.32	2.09
February	0.80	0.58	2.15	1.99	1.75
March	1.30	0.73	3.00	2.80	2.50
April	1.45	0.75	3.19	2.99	2.68
May	0.85	0.50	2.01	1.88	1.67
June	1.40	0.79	3.24	3.02	2.70

Table 8.5: Normally distributed resignation data and critical values

Unlikely scenarios

In light of the uncertainty in the resignation data, the personnel manager needs to hire enough extra trainees to make sure that there are enough flight attendants under most circumstances. It is not economical to cover the unlikely extreme scenario's in which the number of resignations is far beyond the average. Eliminating the extreme scenario's can be accomplished through the use of probabilistic constraints.

Probabilistic constraints

Consider a slightly modified version of the flight attendant balance constraint, in which the expression on the left can be interpreted as the exact number of flight attendants that can leave without causing a shortage or surplus.

$$x_{t-1} - y_{t-2} - x_t = l_{t-1} \qquad \forall t$$

By aiming for a surplus, it is possible to avoid a shortage under most resignation scenarios. The flight attendant balance constraint can be altered into any of the following two equivalent probabilistic constraints:

$$\begin{array}{lcll} \Pr[x_{t-1} - y_{t-2} - x_t >= l_{t-1}] & \geq & 1 - \alpha & \forall t \\ \Pr[x_{t-1} - y_{t-2} - x_t <= l_{t-1}] & \leq & \alpha & \forall t \end{array}$$

The value of α is assumed to be small, indicating there is frequently a surplus (the first form) or there is infrequently a shortage (the second form).

Deterministic equivalence

As explained in Section 6.6, both probabilistic constraints have the same deterministic equivalent, namely

$$x_{t-1} - y_{t-2} - x_t \geq \bar{l}_{t-1} \qquad \forall t$$

where $\bar{l}_t$ could be one of the critical values from Table 8.5 depending on the level of α selected by management.

Summary of new model

The new model with the deterministic equivalent of the probabilistic constraints can now be summarized as follows.

Minimize:

$$\sum_t cx_t + d(y_t + y_{t-1})$$

Subject to:

$$\begin{aligned} x_t &\le x_{t-1} - \bar{l}_{t-1} + y_{t-2} && \forall t \\ ux_t + v(y_t + y_{t-1}) &\ge r_t && \forall t \\ x_t, y_t &\ge 0, \quad \textit{integer} && \forall t \end{aligned}$$

Integer requirement

This new model strongly resembles the model in Section 8.2. Note that the parameter values of $\bar{l}_t$ are likely to be fractional in the above version. This implies that the integer requirement on both x_t and y_t are necessary, and that the above balance constraints for flight attendants are likely not to be tight in the optimal solution.

8.6 Summary

In this chapter, a multi-period planning model was developed to determine the hiring and training requirements for an airline company. Since the number of new flight attendants must be integer, the resulting model is an integer programming model. For large data sets, one option to obtain an integer solution is to round an LP solution. If you understand the model, it is possible to develop a sensible rounding procedure. Such a procedure might be considerably more efficient than using an integer programming solver. Towards the end of the chapter, the uncertainty in the number of flight attendants resigning during the planning interval was modeled. By modifying the resulting probabilistic constraints to deterministic constraints, an ordinary integer optimization model was found.

Exercises

8.1 Implement the mathematical program described at the end of Section 8.2 using the example data provided in Section 8.1 . Verify that the optimal integer solution produced with Aimms is the same as the solution provided in Table 8.2.

8.2 Solve the mathematical program as a linear program (by using `rmip` as the mathematical program type), and implement the rounding heuristic described in Section 8.4. Verify that your findings coincide with the numbers displayed in Table 8.4.

8.3 Extend the mathematical program to include probabilistic constraints using the observed distribution data as described in Section 8.5 and compare the results for the different α-values as presented in Table 8.5.

Bibliography

[Ch83] V. Chvátal, *Linear programming*, W.H. Freeman and Company, New York, 1983.

[Ep87] D.D. Eppen, F.J. Gould, and C.P. Schmidt, *Introductory management science*, 2nd ed., Prentice-Hall, Englewood Cliffs, N.J., 1987.

[Wa75] H.M. Wagner, *Principles of operations research*, 2nd ed., Prentice-Hall, Englewood Cliffs, N.J., 1975.